Multivariable Calculus

Exercises

1. Calculate all partial derivative of the following functions:
   - $g(x, y) = (x - y)^2$
   - $f(x, y) = \sin(\ln(x)) + xy + y^2$
2. Calculate the gradient of the following functions:
   - $f(x_1, x_2) = \dfrac{\sqrt{x_1^2 - 1}}{x_2^4}$
   - $g(x, y, z) = (xyz - x^2 + y^2 - z^2)^2$
3. Given the following functions f and directions $\mathbf{u}$. Calculate the directional derivative at the given coordinates.
   - $f(x, y) = x^2 + \sqrt{xy}$ and $\mathbf{u} = (\frac{1}{2}, \frac{1}{2})$ at position $(\sqrt{2}, 2\sqrt{2})$
   - $f(x, y, z) = x + y + z$ and $\mathbf{u} = (2, 0, 1)$ at position (π, π^2, π^3)
4. Calculate the following multiple integrals:
   - $\displaystyle\int_1^2 \int_1^2 \left(x^2 + xy^2\right) dydx$
   - $\displaystyle\int_0^{\pi} \int_{-\pi/2}^{\pi/2} \int_2^4 \left(z^3 \sin(y) \cos(x)\right) dzdydx$
5. Find the stationary points of the following functions:
   - $f(x, y) = \sin(x) \sin(y)$ on the domain $[-\pi, \pi] \times [-\pi, \pi]$
   - $f(x, y) = x^2 y - xy^2 + 5$
   - $f(x, y, z) = x^2 + y^2 + z^2 - 4x + 8y - 6z + 29$ and also determine the type of the stationary point(s).